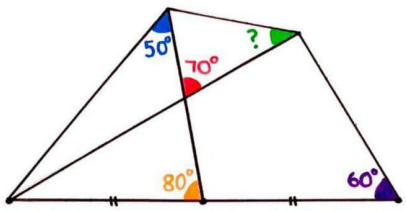
October Math Gems

Problem of the week 2

§1 problems

Problem 1.1. Find the unknown value.



Problem 1.2. Find the maximum obtainable value by adding a single set of parentheses into the expression

$$1 + 2 \times 3 + 4 \times 5 + 6$$

Problem 1.3. Evaluate the following expression

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{2016}+\sqrt{2017}}$$

Problem 1.4. Evaluate

$$\frac{e}{\sqrt{e}} \times \frac{\sqrt[3]{e}}{\sqrt[4]{e}} \times \frac{\sqrt[5]{e}}{\sqrt[6]{e}} \times \ldots$$

Where e is the euler number

Problem 1.5. What is the domain of the following function?

$$\frac{x+2}{x^2+x-2}$$

Problem 1.6. If the length of a rectangle is decreased by 30 percent, and the width is increased by 15 percent. The area of the new rectangle will be smaller than the original rectangle by what percent?

Problem 1.7. The side lengths of a scalene triangle are the roots of the polynomial

$$x^3 - 20x^2 + 131x - 281.3.$$

Find the square of the area of the triangle.

Problem 1.8.

$$\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$$

Solve for x.

Problem 1.9. If $a^2 + 1 = 9a$, then compute the value of the following expression

$$a^2 + \frac{1}{a^2}$$

Problem 1.10. If $4b^2 + \frac{1}{b^2} = 2$, then compute the value of the following expression

$$8b^3 + \frac{1}{b^3}$$

Problem 1.11. Solve for x

$$x^2 + \frac{9x^2}{(x+3)^2} = 27$$

Problem 1.12. If $x^2 + y^2 - 2x + 6y + 10 = 0$, then compute the value of the following expression

$$(x^2 + y^2)$$

is?

Problem 1.13. Compute

$$\frac{5+\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{7+\sqrt{12}}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{63+\sqrt{992}}{\sqrt{31}+\sqrt{32}}$$

Problem 1.14. If $3a + 4b = 5$, then what is the minimum value of $a^2 + b^2$?

Problem 1.15. Evaluate

$$\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + ... + \frac{1}{\sqrt{167}+\sqrt{169}}$$

Problem 1.16. Let a, b and c are distinct nonzero real numbers where,

$$a + \frac{1}{a} = b + \frac{1}{b} = c + \frac{1}{c}$$

Then, find the value of $(abc)^{2022}$

Problem 1.17. Compute

$$\log_{a^a} b^b \times \log_{b^c} c^a \times \log_{c^b} a^c$$

Problem 1.18. If $x, y, z > 0$ such that $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$
Compute

$$x^x y^y z^z$$

Problem 1.19. Solve the following equation

$$x^{2\ln(x)} = 10x^2$$

Problem 1.20. Compute

$$\log_5 \sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5....\infty}}}}}$$